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Multichannel Kondo screening in a one-dimensional correlated electron system

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We present the exact *Bethe Ansatz* solution of a multichannel model of one-dimensional correlated electrons coupled antiferromagnetically to a magnetic impurity of arbitrary spin S. The solution reveals that interactions in the bulk make the magnetic impurity drive both spin *and* charge fluctuations, producing a mixed valence $n_{imp} \neq 0$ at the impurity site, with an associated effective spin $S_{eff} = S + |n_{imp}|/2$ in the presence of a magnetic field. The screening of the impurity is controlled by the size of the impurity spin independently of the number of channels, in contrast to the multichannel Kondo effect for free electrons.

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Introduction. - Impurities play an essential role in correlated electron systems, in particular in 1D where even a small amount of defects may drastically change the properties of the system. While the effect of a single potential scatterer is by now fairly well understood [1], the case of a *dynamical* scatterer in a 1D interacting electron system - like a magnetic impurity - largely remains an open problem [2].

Of particular interest is to understand what happens when the electrons are coupled to a local magnetic moment (of magnitude S) in several degenerate channels m. In the case of a 3D Fermi liquid, the impurity induces strong correlations among the electrons, with the low-temperature physics depending on the relation between the impurity spin and the number of electron channels [3–7]. It is natural to ask whether the correlations inherent in a 1D electron system (*Luttinger liquid,* [8]) will modify this behavior. Does novel effects appear, or does one recover the same multichannel Kondo physics as for free electrons in 3D? Apart from the possible experimental relevance of these questions to the study of artificial impurities in mesoscopic devices [9], their resolution is an interesting issue in its own right. A study is also motivated by the recent interest in the Kondo effect in the high-T_c cuprates [10]: it is well known that at least two orbitals, $3d_{x^2-y^2}$ and $3d_{z^2}$, play an essential role there, so the Kondo effect should have multichannel nature. An analysis of the simpler, analog problem in 1D - with *correlated* electrons - may provide important clues for how to model the effect in the cuprates.

In this Letter we attack the problem by considering an *integrable* model of a magnetic impurity embedded into a multichannel interacting electron system. We here take a multichannel extension of the supersymmetric $t-J$ model in 1D [11], with the electrons coupled to a localized spin (of magnitude S) by an antiferromagnetic exchange interaction that preserves integrability. This makes applicable a *Bethe Ansatz* approach, allowing us to obtain *exact* results for the groundstate properties as well as the finite temperature behavior. Our solution reveals that there are two distinct processes that govern the zero temperature response due to the impurity: First, the singlet Cooper-like pairs of electrons present in the ground state break up and get temporarily trapped when scattering off the impurity, thus producing an impurity valence $n_{imp} \neq 0$ with an associated effective impurity spin $S_{eff} = S+|n_{imp}|/2$ in the presence of a magnetic field H. Secondly, this effective composite spin gets screened by unbound electrons excited by H from the spin singlet groundstate. The type of low-temperature behavior that emerges depends only on the size of the bare impurity spin S, and is insensitive to the number of channels. This property is different from that of the ordinary multichannel Kondo effect in a Fermi liquid.

The model. - The Hamiltonian $\mathcal{H}_{host}$ of the multichannel extension of the supersymmetric $t-J$ model can be written as $\mathcal{H}_{host} \equiv \sum_j \mathcal{H}_{j,j+1}$, with

$$\mathcal{H}_{j,j+1} = -\mathcal{P}(c^\dagger_{j,\sigma,f}c_{j+1,\sigma,f} + h.c)\mathcal{P} - c^\dagger_{j,\sigma,f}c_{j,\sigma,f'}c^\dagger_{j+1,\sigma',f'}c_{j+1,\sigma',f} + c^\dagger_{j,\sigma,f}c_{j,\sigma',f}c^\dagger_{j+1,\sigma',f'}c_{j+1,\sigma,f'} \ . \qquad (1)$$

Here $j = 1, ..., L$ is a site index, $\sigma = \pm$ denotes the spin projection, $f = 1, \ldots, m$ is a *flavor* quantum number indexing the available electron channels, and $\mathcal{P}$ is a projector on the subspace of singly occupied states. All indices are summed over. The scattering matrix for two electrons is given by $\hat{X}(p_i-p_j) = [(p_i-p_j)\hat{I}+i\hat{P}^s]\otimes[(p_i-p_j)\hat{I}-i\hat{P}^f/[(p_i-p_j)^2+1]$ with $p_{i,j}$ the corresponding *charge rapidities*, and $\hat{P}^s$ and $\hat{P}^f$ the permutation operators in spin and flavor subspace, respectively. The $\hat{X}$-matrices satisfy the Yang-Baxter equation $\hat{X}^{12}(p_1-p_2)\hat{X}^{13}(p_1-p_3)\hat{X}^{23}(p_2-p_3) = \hat{X}^{23}(p_2-p_3)\hat{X}^{13}(p_1-p_3)\hat{X}^{12}(p_1-p_2)$, thus ensuring the integrability of the model.

We now insert an additional site on the lattice, call it 0, attach a magnetic impurity to it, and couple it to the electrons on neighboring sites (sites 1 and L, given periodic boundary conditions). To preserve integrability this interaction must be judiciously chosen. We here follow the strategy pioneered in [12] and *define* the impurity-host interaction via an electron-impurity S-matrix $\hat{S}$ that satisfies the Yang-Baxter equation $\hat{X}^{12}(p_1-p_2)\hat{S}^{10}(p_1-p_0)\hat{S}^{20}(p_2-p_0)=\hat{S}^{20}(p_2-p_0)\hat{S}^{10}(p_1-p_0)\hat{X}^{12}(p_1-p_2)$, with p_0 measuring the impurity energy level. This approach assures that the model remains integrable in presence of the impurity. The $\hat{S}$-matrix can still be chosen in a number of ways [13], and here we take it to be similar to that of the multichannel Kondo problem in a free electron gas [7] and let it act nontrivially only in the spin subspace. Writing out the components,

$$\hat{S}^{\alpha\alpha'}_{MM'}(p)=a(p)\frac{(p+i/2)\delta_{\alpha\alpha'}\delta_{MM'}+i\sigma^k_{\alpha\alpha'}S^k_{MM'}}{p+i}\ , \tag{2}$$

with σ^k the Pauli matrices ($k=x,y,z$), S^k the impurity spin matrices, $|M|\leq S$ the component of the impurity spin S (with unprimed/primed indices referring to incoming/outgoing states), and $a(p)\equiv[(p^2+1)/(p^2+(S+1/2)^2)]^{1/2}$. The corresponding impurity Hamiltonian $\mathcal{H}_{imp}$ can be written in the form:

$$\begin{aligned}\mathcal{H}_{imp}=&J\big(\mathcal{H}_{L,0}+\mathcal{H}_{0,1}+\{\mathcal{H}_{L,0},\mathcal{H}_{0,1}\}\big)+(1-3S(S+1)J-\frac{J}{4})\mathcal{H}_{L,1}\\&+2p_0J[(\mathcal{H}_{L,0}+\mathcal{H}_{0,1}),\mathcal{H}_{L,1}]\ .\end{aligned} \tag{3}$$

Here $\mathcal{H}_{0,1}$ and $\mathcal{H}_{L,0}$ are generalized permutation operators of a particle with spin S but no flavor (impurity) and an electron (carrying both spin *and* flavor), while $\mathcal{H}_{L,1}$ is defined in Eq. (1). The function $J=[p_0^2+(S+\frac{1}{2})^2]^{-1}$ plays the role of an effective exchange constant between the impurity site and neighboring sites. We should mention that the structure of $\mathcal{H}_{imp}$ for the periodic chain in (3) simplifies considerably when the impurity is located at the edge of an *open* chain, with the impurity connected to the host via only one link, with coupling constant J. It is here worth pointing out that the low-energy behavior of impurities in integrable open and periodic chains of correlated electrons are qualitatively the same, as was shown recently in [14]. We should also point out that our approach is different from that recently advocated by Wang and Voit in their study of the ferromagnetic Kondo effect in a Luttinger liquid, where the impurity is simulated by a boundary potential [15]. In contrast, in our formulation the full dynamics of the impurity is retained.

Bethe Ansatz equations. - The model thus constructed can be diagonalized exactly by an *algebraic Bethe ansatz* [16]. The procedure is rather cumbersome, and we here only give the result. The eigenstates are characterized by $m+2$ sets of quantum numbers: The *charge rapidities* $\{p_j\}_{j=1}^N$ (with N the total number of electrons), the *spin rapidities* $\{\lambda_\alpha\}_{\alpha=1}^M$ (M counting the number of spin-down electrons), and the m sets of *flavor rapidities* $\{\mu_\beta\}_{\beta=1}^{m^{(i)}}, i=1,2,...,m$ (with $m^{(i)}=\sum_{k=i+1}^m n^{(k)}$, $n^{(k)}$ counting the number of electrons in channel $k, k=1,2,...,m$). Each state corresponds to a particular solution of the nested *Bethe Ansatz* equations

$$\begin{aligned}&\prod_{\tau=\pm}\prod_{\beta=1}^{m^{(k+\tau)}}e_1(\mu^{(k)}_\alpha-\mu^{(k+\tau)}_\beta)=\prod_{\gamma=1}^{m^{(k)}}e_2(\mu^{(k)}_\alpha-\mu^{(k)}_\gamma)\ ,\\&[e_{2S+1}(p_j-p_0)e_2(p_0-p_j)]^{1/2}e_1^L(p_j)=\prod_{\alpha=1}^{M}e_1(p_j-\lambda_\alpha)\prod_{\beta=1}^{m^{(1)}}e_1(\mu^{(1)}_\beta-p_j)\\&e_{2S}(\lambda_\alpha-p_0)\prod_{j=1}^{N}e_1(\lambda_\alpha-p_j)=\prod_{\beta=1}^{M}e_2(\lambda_\alpha-\lambda_\beta)\ ,\end{aligned} \tag{4}$$

where $e_n(x)\equiv(2x+in)/(2x-in)$, $\mu^{(0)}_j=p_j$, $m^{(0)}=N$, $m^{(m+1)}=0$, and L is the number of lattice sites (not counting the impurity site). We shall assume that no external fields couple to the electron channels, and can hence confine our attention to the flavor-singlet subspace. The energy (up to an additive constant) and the magnetic moment are then equal to $E=\sum_j^N(p_j^2+(1/4))^{-1}$ and $S^z=N/2+S-M$, respectively.

The thermodynamics of the model is described by the usual string hypothesis [17]. In the thermodynamic limit, with $L,N,M,m^{(j)}\to\infty$, their ratios being fixed, the model has the following possible excitations: (i) unbound electrons with charge rapidities p_j; (ii) spin singlet Cooper-like pairs with $p_j=\lambda_j\pm i/2$; (iii) spin strings (bound states of any number of "down spins"); and (iv) flavor strings for each channel. (Since the host interactions in (1) carry opposite signs in spin- and flavor subspaces, bound states of different channels as well as bound states of electrons

and spin strings are suppressed.) Introducing distribution functions *(densities)* for particles and holes for each class of excitations we can write down the corresponding thermodynamic *Bethe ansatz* equations. Then, by minimizing the free energy, we extract the integral equations for each excitation class. These equations have the same structure as those of the multichannel Kondo problem for free electrons [19], and differ only in the driving terms (which are independent of energy and density). This implies that in the high-temperature limit, where the driving terms are unimportant, the effect of our impurity is similar to that of a Kondo impurity in a multichannel free electron gas.

Groundstate properties. - Let us focus instead on the groundstate properties of the impurity, and study how it depends on the host band filling and an applied magnetic field. In the zero temperature limit, $T \to 0$, only excitations of classes (i), (ii), and (iv) (for unbound *flavorons* and pairs of them, i.e. flavor strings of length 1 and 2 for each channel) can have negative energies. Thus, the filling of these states is determined by the Dirac seas of the groundstate of the model. In the singlet flavor sector we can solve the equations for densities and energies of the flavorons as functions of the densities and energies of unbound electrons and singlet spin-charge pairs. As a result, we obtain the ground state equations for unbound electrons and pairs:

$$\begin{aligned} \rho_h(p)+(1-a_1\star s_1)\star(\rho(p)+a_1\star\sigma(\lambda))&=a_1(p)+\tfrac{1}{2L}[a_{2S+1}-a_2](p-p_0)\ , \\ \sigma_h(\lambda)+(1+a_2)\star(1-a_1\star s_1)\star(\sigma(\lambda)+s\star\rho(p))&=a_2(\lambda)+\tfrac{1}{2L}[a_{2S+2}-a_{2S}-a_3-a_1](\lambda-p_0)\ , \end{aligned} \tag{5}$$

where $\star$ denotes convolution, $\rho(\rho_h)$ and $\sigma(\sigma_h)$ are densities for unbound electrons and pairs, respectively. The integration intervals are given by $|p| > B$ and $|\lambda| > Q$, with Q and B playing the role of Fermi points for unbound electrons and pairs, respectively. The Fourier transforms of the kernels a_n, s and s_1 are given by $\exp(-n|\omega|/2)$, $\cosh^{-1}/2\omega$, and $\sinh[(m-1)|\omega|/2]/\sinh(m|\omega|/2)$, respectively. Eqs. (5) are linear in the densities, and the driving terms of the host and the impurity are additive. Separating the densities into bulk and impurity parts then allows us to calculate the valence and the magnetization of the impurity in the groundstate. In the absence of an external magnetic field we have $B = \infty$ (no unbound electrons). The limit $Q \to \infty$ corresponds to a vanishing pair density, while $Q \to 0$ is the limit of $1/2m$-filled bands of the host (no pair holes, corresponding to a vanishing Fermi velocity). For the nonmagnetic groundstate it follows that the effective valence of the impurity varies as a function of electron number from $n_{imp} = 0$ for vanishing pair density to $n_{imp} = -m$ for $1/2m$-filled bands, with the valence measured w.r.t. the groundstate of spin-paired electrons (i.e. its negative sign indicates an excess of pair *holes* due to the presence of the impurity). This unusual behavior is caused by the host interaction in (1) which supports a groundstate with a Dirac sea of bound singlet Cooper-like electron pairs, not present in the free electron gas.

Of special interest is the behavior of the impurity magnetization S^z_{imp}. With no magnetization in the bulk, we have $S^z_{imp} = S$. By turning on a magnetic field, the number of unbound electrons increases while the number of singlet pairs decreases (as required by electron number conservation). By eliminating the pair density $\sigma(\lambda)$ from the second equation in (5), we obtain the Fredholm equation

$$\rho_h(p)+\rho(p)-F\star\rho = s\star\sigma_h+s(p)+\tfrac{1}{2L}s\star a_{2S}(p-p_0)\ , \tag{6}$$

where $F(\omega) = 1-\tanh(|\omega|/2)[1-\exp(-m|\omega|)]^{-1}$, and the integration over the pair hole density is over the finite interval $[-Q, Q]$. This yields an explicit connection between the densities of unbound particles (electrons or holes) and spin singlet pair holes. As the Zeeman splitting is typically much smaller than the Fermi energy, we can neglect the influence of the pairs on the impurity magnetization as long as the magnetic field is sufficiently weak. For this case ($H \ll 1$), choosing $Q = 0$ (i.e. $1/2m$-filled bands) and assuming that $|p_0| \gg B$, we can solve Eq. (6) exactly by reducing it to a sequence of coupled Wiener-Hopf integral equations. In this way we obtain *two* distinct regimes for the behavior of S^z_{imp} with magnetic field. If $S > 1/2$ the impurity spin becomes asymptotically free:

$$S^z_{imp} = \mu\big[1 \pm \tfrac{m}{2}(|\log(H/T_H)|)^{-1} + \ldots\big]\ , \quad S > \frac{1}{2} \tag{7}$$

with $T_H = 2\pi(\frac{m}{2e})^{m/2}T_K/\Gamma(\frac{m}{2})$, and $T_K \propto \exp(-\pi p_0)$ playing the role of the Kondo temperature. When $H \ll T_H$, $\mu = S$, and the upper sign in (7) gets selected. On the other hand, with $T_H \ll H \ll 1$, $\mu = S + m/2$ and the lower sign in (7) is selected. Note that for this case H is still much smaller than the spin saturation field in the bulk (corresponding to a transition to a ferromagnetic bulk state). Also note that the *underscreened* behavior in (7) for $H << T_H$ appears only for $m < 2S$ in a free electron gas [7], in contrast to the correlated host studied here. For the *exactly screened case* $S = 1/2$,

$$S^z_{imp} \sim \frac{mH}{2\pi T_K}, \quad S = \frac{1}{2}, \tag{8}$$

producing a finite zero-field magnetic susceptibility. Again, this is different from a free electron host where this behavior sets in for $m = 2S$. Most importantly, the case of *overscreening* ($m > 2S$ for a free host, with critical scaling of the impurity magnetization) *does not exist* in this interacting electron system. Thus, electron correlations in the host due to direct electron-electron interaction here suppresses the critical overscreened behavior of a Kondo-like impurity, similar to what happens in the presence of a channel anisotropy for free electrons [18]. It is important to realize that by increasing the magnetic field we deplete the number of singlet pairs. Therefore, as we have already seen, by varying the field we can smoothly tune the effective impurity valence n_{imp} from $-m$ to 0. In the underscreened case this means that the uncompensated part of the *effective* impurity spin $S_{eff} = S + |n_{imp}|/2$ decreases with increasing field, and recovers its zero-field value S at the spin-saturation field.

Low-temperature thermodynamics. - We now turn to the low-temperature thermodynamics. As we have already mentioned, the thermodynamic Bethe ansatz equations differ from those for the free electron gas multichannel Kondo problem only in the driving terms, and we can hence use an analysis similar to that in [4,19]. The main idea is to rewrite the $T \to 0$ thermodynamic Bethe ansatz equations for *all* excitations on universal form. This can be realized in the "Kondo limit" (with suppressed charge fluctuations), yielding for the universal energy potentials ϕ_j [19]:

$$\phi_j(\lambda) = s \star \log(1 + e^{\phi_{j-1}})(1 + e^{\phi_{j+1}}) - \delta_{j,m} e^{\pi\lambda} \ , \tag{9}$$

with $\lim_{j\to\infty}(\phi_j/j) = H/T$. It follows that in this limit the impurity low-temperature free energy can be written as a function of ϕ_m :

$$F_{imp} = -T \int_{-\infty}^{\infty} \frac{d\lambda a_{2S}(\lambda) \log(1 + e^{\phi_m})}{2\cosh[\pi\lambda + \log(T/T_K)]} \ . \tag{10}$$

The qualitative behavior of F_{imp} is independent of the relative values of m and S, in analogy with the groundstate properties. In the underscreened case $S > 1/2$, the impurity magnetic susceptibility χ_{imp} shows Curie-like temperature dependence while the specific heat C_{imp} exhibits a Schottky anomaly at $T \sim H$ and a Kondo resonance. For the exactly screened case $S = 1/2$ local Fermi liquid behavior holds: $\chi_{imp} \approx m/2\pi T_K$, $C_{imp} \approx \pi m T/(m+2)T_K$.

Discussion. - To conclude, we have found an exact *Bethe Ansatz* solution of the problem of a Kondo-like antiferromagnetic impurity embedded into the multichannel supersymmetric $t-J$ model. Our solution reveals that there are two mechanisms governing the magnetic behavior of the impurity: holes in the distribution of Cooper-like pairs increase the effective spin of the impurity, while unbound electrons screen this effective value. Note, that for the $t-J$ model there is no spin gap, so the unbound electrons and Cooper-like pairs coexist even for small external magnetic fields. In contrast to the multichannel Kondo effect in a Fermi liquid, the screening with unbound electrons are realized in only *two* different ways independent of the number of channels, and controlled only by the value of the *bare* impurity spin: *underscreening*, with an asymptotically free effective impurity spin, and *exact screening* with a finite magnetic susceptibility of the impurity. Overscreened critical behavior is absent for this correlated electron model. As required by integrability, our model supports only forward scattering of the electrons off the impurity, and hence does not produce localization of the electrons. In contrast, more realistic 1D magnetic impurity models must contain also back scattering terms which are expected to renormalize to an effective boundary potential, in addition to producing an impurity thermal response sensitive to the electron-electron interaction [20]. Yet, our model dramatically shows how correlations in the bulk make the magnetic impurity drive both spin *and* charge fluctuations. This property is expected to be generic and our exact solution provides a detailed picture of its possible realization.

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